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Quantitative Performance Robustness Linear Quadratic Optimal System Design

Chih-Min Lin* and Ze-Rong Shi†
Yuan-Ze Institute of Technology, Chung-Li, 320 Taiwan, Republic of China

The synthesis methodology of quantitative performance robustness linear quadratic optimal control is developed. Wiener–Hopf linear quadratic optimal control and quantitative feedback theory are applied to achieve the quantitative performance robustness optimal control design. Since the internal stability is satisfied, this design method performs appropriately for any stable or unstable, minimum- or nonminimum-phase system. This is the generalized result of some previous work. Two frequency-dependent weighting functions are shaped and two-degree-of-freedom compensators are designed to achieve the robust optimal control in the presence of plant uncertainty. Moreover, for the minimum-phase plant, by appropriately factorizing the two-degree-of-freedom compensators, the feedback loop can be designed to satisfy the return difference equality. The F-4E flight control system is considered as the design example to illustrate the design algorithm.

I. Introduction

THE linear quadratic optimal control design has been widely investigated in the past, both in the time and the frequency domains. In the time domain, the linear quadratic regulator (LQR) optimal control problem satisfies the return difference equality, so that it possesses the stability properties with gain margin ∞, gain reduction $\leq \frac{1}{2}$, and phase margin $\geq$ 60 deg (Ref. 1). In the frequency domain, Wiener–Hopf linear quadratic optimal design has been discussed thoroughly in Ref. 2; however, the plant imbedded is restricted to be stable and minimum phase. Youla et al.,[3] Youla and Bongiorno,[4] and Grimble[5] have presented the parametric controller to ensure internal stability for the closed-loop system and to achieve Wiener–Hopf linear quadratic optimal control. Recently, several papers have paid attention to the robust linear quadratic optimal control design via the time-domain approach[6,7] and the frequency-domain approach[8,9]; however, these results are the stability robustness problem; they cannot achieve the desired quantitative performance robustness. The quantitative feedback theory (QFT) developed by Horowitz and Sidi[10] is known to be a very effective design tool for feedback systems with plant uncertainty. In Ref. 11, the QFT technique is employed for the robust Wiener–Hopf optimal design; however, the plant is restricted to be stable and minimum phase. In Ref. 12 the plant can be relaxed to be nonminimum phase but still has to be stable because unstable plant inner-loop feedback is needed. Since the optimal systems designs[2–9,11,12] do not satisfy the return difference equality, these systems do not possess the stability robustness properties as do time-domain LQR optimal systems.

In this paper, the synthesis methodology of quantitative performance robustness linear quadratic optimal control is developed. Wiener–Hopf linear quadratic optimal control is introduced and is then incorporated with the QFT robust control design technique to achieve the quantitative performance robustness optimal control design. Since the internal stability is satisfied, this design algorithm performs appropriately for any stable or unstable, minimum- or nonminimum-phase system. It is shown that the previous works[2,11,12] are special cases of this generalized result. Two frequency-dependent weighting functions are shaped, and two-degree-of-freedom compensators are designed to achieve this quantitative performance robustness optimal control in the presence of plant uncertainty. Moreover, for the minimum-phase plant, by appropriately factorizing the two-degree-of-freedom compensators, the feedback loop can be designed to satisfy the return difference equality, i.e., the designed system can also possess the stability margins as in time-domain LQR optimal systems.

The notations used in this paper are standardized: $\alpha^*(s)$ is the complex conjugate of the transfer function $\alpha(s)$. For a transfer function $\beta(s)$ is expressed as

$$\beta(s) = \{\beta(s)\}^+\{\beta(s)\}^- \tag{1}$$

where $\{\beta(s)\}^+$ and $\{\beta(s)\}^-$ denote the spectral factorization of $\beta(s)$ with $\{\beta(s)\}^+$ containing all of the poles and zeros in $Re(s) \leq 0$ and $\{\beta(s)\}^-$ containing all of the poles and zeros in $Re(s) \geq 0$. Here

$$\gamma(s) = [\gamma(s)]_+ + [\gamma(s)]_- \tag{2}$$

where $[\gamma(s)]_+$ denotes the part of the partial fraction expansion associated with all poles of $\gamma(s)$ in $Re(s) \leq 0$. Thus, $[\gamma(s)]_+$ is analytic in $Re(s) > 0$; the remainder of the expansion is represented as $[\gamma(s)]$.

II. Wiener–Hopf Linear Quadratic Optimal Control

Consider the single-input single-output system shown in Fig. 1, where $P(s)$, $P_0(s)$, $C(s)$, and $M(s)$ are the uncertain plant, nominal plant, controller, and reference model, respectively.

Considering the nominal case, it is seen that the output signal

$$y(s) = S(s)d(s) + T(s)[r(s) - n(s)] \tag{3}$$

where $r(s)$, $d(s)$, and $n(s)$ are the reference signal, disturbance, and measurement noise, respectively; assume that $r(s)$, $d(s)$, and $n(s)$ are mutually uncorrelated and

$$S(s) \equiv \frac{1}{1 + P_0(s)C(s)} \tag{4}$$

$$T(s) \equiv 1 - S(s) \tag{5}$$

The control input

$$u(s) = C(s)S(s)[r(s) - n(s) - d(s)] \tag{6}$$

The output error

$$e(s) = [M(s) - T(s)]r(s) - S(s)d(s) + T(s)n(s) \tag{7}$$

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*Associate Professor, Department of Electrical Engineering, 135 Far-East Road.
†Graduate Student, Department of Electrical Engineering, 135 Far-East Road.

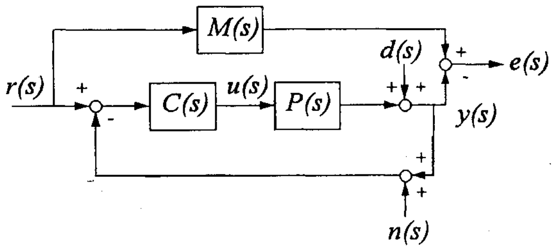

Fig. 1 Feedback control system with reference model.

Let $\phi_{ee}(s)$ and $\phi_{uu}(s)$ be the power spectral densities of $e(s)$ and $u(s)$, respectively, and be given by

$$\phi_{ee}(s) = [M(s) - T(s)]\phi_{rr}(s)[M^*(s) - T^*(s)] + S(s)\phi_{dd}(s)S^*(s) + T(s)\phi_{nn}(s)T^*(s) \tag{8}$$

$$\phi_{uu}(s) = C(s)S(s)\Phi(s)S^*(s)C^*(s) \tag{9}$$

where $\phi_{rr}(s)$, $\phi_{dd}(s)$, and $\phi_{nn}(s)$ are the power spectral densities of $r(s)$, $d(s)$, and $n(s)$, respectively, and

$$\Phi(s) \equiv \phi_{rr}(s) + \phi_{dd}(s) + \phi_{nn}(s) \tag{10}$$

The linear quadratic cost function J is expressed as

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}[Q(s)\phi_{ee}(s) + R(s)\phi_{uu}(s)]\,\mathrm{d}s, \qquad s = j\omega \tag{11}$$

where the weighting function $Q(s) \geq 0$ and $R(s) \geq 0$ for all imaginary s (i.e., $s = j\omega$).

Substituting Eqs. (8) and (9) into Eq. (11) yields

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\Big(Q(s)\big\{[M(s) - T(s)]\phi_{rr}(s)[M^*(s) - T^*(s)] + S(s)\phi_{dd}(s)S^*(s) + T(s)\phi_{nn}(s)T^*(s)\big\} + R(s)C(s)S(s)\Phi(s)S^*(s)C^*(s)\Big)\,\mathrm{d}s \tag{12}$$

Let

$$P_0(s) = B(s)/A(s) \tag{13}$$

where $A(s)$ and $B(s)$ constitute any coprime, proper, stable, rational decomposition of $P_0(s)$. We now select proper, stable, rational functions $U(s)$ and $V(s)$ such that

$$A(s)V(s) + B(s)U(s) = 1 \tag{14}$$

Lemma 1 (Ref. 3). $S(s)$ is internally stable if and only if we have

$$S(s) = A(s)[V(s) + K(s)B(s)] \tag{15}$$

i.e.,

$$T(s) = 1 - S(s) = B(s)[U(s) - K(s)A(s)] \tag{16}$$

and the corresponding controller is given by

$$C(s) = \frac{U(s) - K(s)A(s)}{V(s) + K(s)B(s)} \tag{17}$$

where $K(s)$ is any rational function, analytic in $Re(s) \geq 0$, and satisfying the constraint $V(s) + K(s)B(s) \not\equiv 0$.

From Eqs. (15) and (17), it is shown that

$$C(s)S(s) = A(s)[U(s) - K(s)A(s)] \tag{18}$$

Substituting Eqs. (15), (16), and (18) into Eq. (12), we have

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} Z(s)\,\mathrm{d}s \tag{19}$$

where

$$Z(s) = Q(s)\Big(\{M(s) - B(s)[U(s) - K(s)A(s)]\}\phi_{rr}(s) \times \{M(s) - B(s)[U(s) - K(s)A(s)]\}^* + [V(s) + K(s)B(s)] \times A(s)\phi_{dd}(s)A^*(s)[V(s) + K(s)B(s)]^* + B(s)[U(s) - K(s)A(s)]\phi_{nn}(s)[U(s) - K(s)A(s)]^*B^*(s)\Big) + R(s)[U(s) - K(s)A(s)]A(s)\Phi(s)A^*(s)[U(s) - K(s)A(s)]^* \tag{20}$$

The problem of minimizing J is to vary over all stable, rational function $K(s)$ that satisfy the restriction $V(s) + K(s)B(s) \not\equiv 0$.

Let

$$A(s) = a_r(s)A_1(s) \tag{21}$$

where $a_r(s)$ absorbs all zeros of $A(s)$ in $Re(s) \geq 0$ (i.e., the unstable poles of the plant).

Perform the spectral factorizations

$$\Delta_1(s)\Delta_1^*(s) = B(s)Q(s)B^*(s) + A(s)R(s)A^*(s) \tag{22}$$

$$\Delta_2(s)\Delta_2^*(s) = A(s)\Phi(s)A^*(s) \tag{23}$$

where $\Delta_1(s)$ and $\Delta_2(s)$are free of poles and zeros in $Re(s) > 0$.

Then we have the following theorem.

Theorem 1. In Fig. 1, for given $P_0(s)$, $M(s)$, $Q(s)$, $R(s)$, $\phi_{dd}(s)$, $\phi_{nn}(s)$, and $\phi_{rr}(s)$, and solving for any two proper, stable, rational functions $U(s)$ and $V(s)$ that satisfy Eq. (14), then the closed-loop transfer function $\hat{T}(s)$ that minimizes J of Eq. (11) is given by

$$\hat{T}(s) = B(s)[U(s) - \hat{K}(s)A(s)] \tag{24}$$

or equivalently, the transfer function of the optimal controller is

$$\hat{C}(s) = \frac{U(s) - \hat{K}(s)A(s)}{V(s) + \hat{K}(s)B(s)} \tag{25}$$

where

$$\hat{K}(s) = \frac{1}{a_r(s)}\left\{\frac{1}{\Delta_1(s)}\left(\hat{F}(s) - \left[a_r(s)\frac{B^*(s)}{\Delta_1^*(s)}Q(s)[M(s)\phi_{rr}(s) + \phi_{dd}(s)]\frac{A^*(s)}{\Delta_2^*(s)}\right]_+\right)\frac{1}{\Delta_2(s)}\right\} + \frac{U(s)}{A(s)} \tag{26}$$

and $\hat{F}(s)$ is a real polynomial determined by the requirement of the finiteness of the cost function J and analyticity of $\hat{K}(s)$ in $Re(s) \geq 0$.

Proof. Refer to Ref. 3.

Remark. Since the internal stability is satisfied, this linear quadratic optimal control design performs appropriately for any stable or unstable, minimum or nonminimum-phase system.

Substituting Eq. (26) into Eq. (24), we have

$$\hat{T}(s) = \frac{-A(s)B(s)}{a_r(s)\Delta_1(s)\Delta_2(s)}\left(\hat{F}(s) - \left[a_r(s)\frac{B^*(s)}{\Delta_1^*(s)}Q(s) \times [M(s)\phi_{rr}(s) + \phi_{dd}(s)]\frac{A^*(s)}{\Delta_2^*(s)}\right]_+\right) \tag{27}$$

In the following, we will discuss some special cases of this optimal system.

Case 1

If $P_0(s)$ is stable [i.e., $a_r(s) = 1$ and $\hat{F}(s) = 0$], $n(s) = 0$ and $d(s) = 0$, and let $\phi_{rr}(s) = Z(s)Z^*(s)$ and $Q(s) = 1$, $P_0(s) = N^*(s)P_1(s)$, where $N^*(s) = \Pi_i(-s + z_i)$, and the z_i are zeros of $P_0(s)$ in $Re(s) > 0$ [i.e., $N^*(s)$ denotes the nonminimum-phase zeros of the plant], then Eq. (27) is reduced to

$$\hat{T} = \frac{N^*(s)}{Y(s)Z(s)N(s)}\left[\frac{Z(s)M(s)N(s)}{Y^*(s)N^*(s)}\right]_+ \tag{28}$$

where

$$Y(s) = \left\{1 + \frac{R(s)}{P_0(s)P_0^*(s)}\right\}^+, \qquad Y^*(s) = \left\{1 + \frac{R(s)}{P_0(s)P_0^*(s)}\right\}^-$$

This special case is the result of the optimal system introduced in Ref. 12.

Case 2

If $P_0(s)$ is stable and minimum phase and $n(s) = 0$, $d(s) = 0$, $M(s) = Q(s) = 1$, and $R(s) = \Gamma^2$, where Γ is a constant real number $\phi_{rr}(s) = Z(s)Z^*(s)$, then Eq. (28) is further reduced to

$$\hat{T} = \frac{1}{Y(s)Z(s)}\left[\frac{Z(s)}{Y^*(s)}\right]_+ \tag{29}$$

where

$$Y(s) = \left\{1 + \frac{\Gamma^2}{P_0(s)P_0^*(s)}\right\}^+, \qquad Y^*(s) = \left\{1 + \frac{\Gamma^2}{P_0(s)P_0^*(s)}\right\}^-$$

This special case is the result of the optimal system introduced in Refs. 2 and 11.

III. Two-Degree-of-Freedom Robust Optimal System

In general, the system of Fig. 1 with the optimal closed-loop transfer function $\hat{T}(s)$ obtained in Eq. (27) does not satisfy the return difference equality. In this section, the equivalent two-degree-of-freedom compensation configuration shown in Fig. 2 is considered so that in addition to maintaining the optimal control of the overall system, the return difference equality of the feedback loop can be also satisfied to possess the stability margins as in time-domain LQR optimal systems.

With this two-degree-of-freedom compensation configuration, the optimal closed-loop transfer function $\hat{T}(s)$ obtained in Eq. (27) can also be expressed as

$$\hat{T} = G(s)\frac{P_0(s)}{1 + D(s)P_0(s)} \equiv G(s)T_1(s) \tag{30}$$

i.e., $T_1(s)$ is the transfer function from $h(s)$ to $y(s)$, and $G(s)$ is the prefilter.

Let

$$X(s) = \hat{F}(s) - \left\{a_r(s)\frac{B^*(s)}{\Delta_1^*(s)}Q(s)[M(s)\phi_{rr}(s) + \phi_{dd}(s)]\frac{A^*(s)}{\Delta_2^*(s)}\right\}^+ \tag{31}$$

then from Eqs. (22), (23), and (27), we have

$$\hat{T}(s)\hat{T}^*(s) = \frac{A(s)A^*(s)B(s)B^*(s)X(s)X^*(s)}{a_r(s)a_r^*(s)\Delta_1(s)\Delta_1^*(s)\Delta_2(s)\Delta_2^*(s)} = \frac{X(s)X^*(s)}{a_r(s)a_r^*(s)\Phi(s)R(s)}\,\frac{P_0(s)R(s)P_0^*(s)}{R(s) + P_0(s)Q(s)P_0^*(s)} \tag{32}$$

If we choose

$$T_1(s)T_1^*(s) = \frac{P_0(s)R(s)P_0^*(s)}{R(s) + P_0(s)Q(s)P_0^*(s)} \tag{33}$$

then

$$R(s) + P_0(s)Q(s)P_0^*(s) = [1 + P_0(s)D(s)]R(s)[1 + P_0(s)D(s)]^* \tag{34}$$

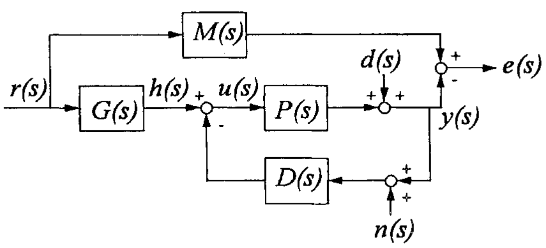

Fig. 2 Two-degree-of-freedom model reference feedback system.

so that the feedback loop of Fig. 2 satisfies the return difference equality with output feedback case. In this case the prefilter will be

$$G(s) = \left\{\frac{X(s)X^*(s)}{a_r(s)a_r^*(s)\Phi(s)R(s)}\right\}^+ \tag{35}$$

which is free of pole and zero in $Re(s) > 0$.

Remarks. 1) Following the discussion in Ref. 1, for the feedback loop to satisfy the return difference equality in Eq. (34), the adopted plant must be minimum phase. This coincides with the fact of time-domain LQR design. 2) In fact, the factorization for $\hat{T}(s) = G(s)T_1(s)$ is not unique. For satisfying the return difference equality of the feedback loop, we have the factorization as in Eqs. (33) and (35). Another factorization, for example, is

$$T_1(s)T_1^*(s) = \frac{P_0(s)P_0^*(s)}{a_r(s)a_r^*(s)\Phi(s)\left[R(s) + P_0(s)Q(s)P_0^*(s)\right]} \tag{36}$$

and then

$$G(s) = \{X(s)X^*(s)\}^+ \tag{37}$$

In this case, the feedback loop of Fig. 2 has not satisfied the return difference equality and the adopted plant can be nonminimum phase.

IV. Quantitative Performance Robustness Linear Quadratic Optimal System

In this section, the QFT is applied to achieve the quantitative performance robustness linear quadratic optimal control design. For the uncertain plant $P(s)$, following the two-degree-of-freedom compensation configuration shown in Fig. 2, it is seen that

$$\hat{T}(s) = \frac{G(s)}{D(s)}\,\frac{L(s)}{1 + L(s)} \tag{38}$$

where $L(s) = D(s)P(s)$ is the loop transfer function of the feedback loop. Because of plant parameter uncertainty, $P(j\omega)$ has template at any specific ω, so that

$$\Delta|L(j\omega)|_{db} = \Delta|P(j\omega)|_{db} \tag{39}$$

When the prefilter is a fixed function for specific ω, then

$$\Delta|\hat{T}(j\omega)|_{db} = \Delta\left|\frac{L(j\omega)}{1 + L(j\omega)}\right|_{db} \tag{40}$$

It is a straightforward matter to follow Horowitz's design algorithm[10] for the derivation of bounds on $L(s)$ in the Nichols chart. By a nominal plant $P_0(s)$, a suitable nominal loop transfer function $L_0(s)$ is chosen. To satisfy the return difference equality, the forbidden region of $L_0(s)$ is shown in Fig. 3. As $L_0(s)$ is determined, the feedback controller is obtained as

$$D(s) = \frac{L_0(s)}{P_0(s)} \tag{41}$$

Since

$$T_1(s)T_1^*(s) = \frac{1}{D(s)D^*(s)}\,\frac{L_0(s)L_0^*(s)}{[1 + L_0(s)][1 + L_0(s)]^*} = \frac{P_0(s)R(s)P_0^*(s)}{R(s) + P_0(s)Q(s)P_0^*(s)} \tag{42}$$

as $L_0(s)$ and $D(s)$ are determined, the corresponding $Q(s)$ and $R(s)$ can be determined. Then the prefilter will be obtained from Eq. (35).

From the preceding analysis, we obtain the following design algorithm for quantitative performance robustness linear quadratic optimal control with the feedback loop satisfying the return difference equality.

1) For the uncertain plant and specified closed-loop performance bound in Eqs. (39) and (40), by the QFT approach, we can determine the desired nominal loop transfer function $L_0(s)$ for a nominal plant $P_0(s)$.

2) From Eq. (41), we obtain the feedback controller $D(s)$.

3) From Eq. (42), we obtain the corresponding weighting functions $Q(s)$ and $R(s)$ for linear quadratic optimal control.

4) From Eq. (35), we obtain the corresponding prefilter.

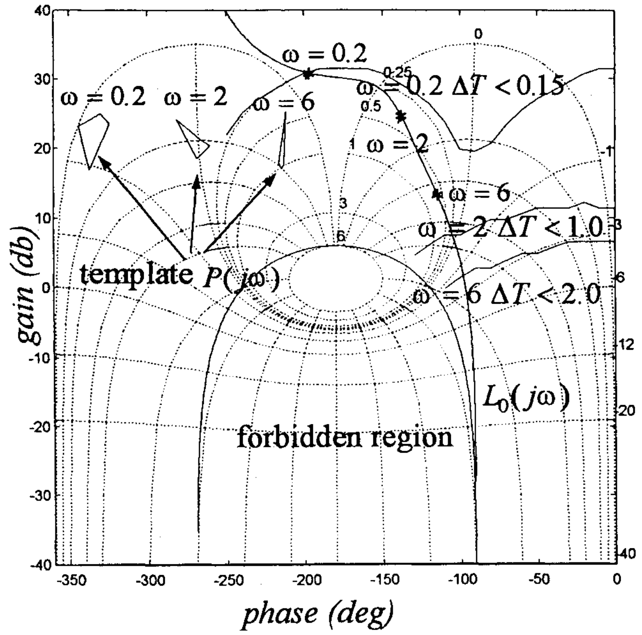

Fig. 3 QFT design in the Nichols chart.

V. Example

In the following, the short period longitudinal mode of the F-4E flight control system is considered as the design example.[13]

The transfer function form of $q(s)/\delta_e(s)$ (pitch rate/deviation of elevator deflection) for four typical flight conditions (FC) can be obtained as follows.

FC 1:

$$P_1(s) = \left.\frac{q(s)}{\delta_e(s)}\right|_{FC1} = \frac{-13.239(s+0.884)}{(s+3.068)(s-1.228)} \tag{43}$$

FC 2:

$$P_2(s) = \left.\frac{q(s)}{\delta_e(s)}\right|_{FC2} = \frac{-36.269(s+1.554)}{(s+4.904)(s-1.784)} \tag{44}$$

FC 3:

$$P_3(s) = \left.\frac{q(s)}{\delta_e(s)}\right|_{FC3} = \frac{-11.308(s+0.637)}{(s+1.878)(s-0.560)} \tag{45}$$

FC 4:

$$P_4(s) = \left.\frac{q(s)}{\delta_e(s)}\right|_{FC4} = \frac{-12.320(s+0.821)}{(s+1.923)(s-0.640)} \tag{46}$$

Here we choose FC 1 as the nominal plant, i.e.,

$$P_0(s) = \frac{-13.239(s+0.884)}{(s+3.068)(s-1.228)} \tag{47}$$

Table 1 Military specifications for flying qualities; see Eq. (49)

Natural frequency, rad/s	FC1	FC2	FC3	FC4
ω_a	2.02	3.05	2.19	2.40
ω_b	7.23	12.6	7.86	8.23

For the model reference optimal control system design, we need to establish the desired reference model. The reference model is chosen to be second order with one zero for reducing the response time, i.e.,

$$M(s) = \frac{\omega_{sp}^2}{z_m}\,\frac{s+z_m}{s^2+2\zeta_{sp}\omega_{sp}s+\omega_{sp}^2} \tag{50}$$

In this reference model, the peak time t_p (in seconds) is

$$t_p = \frac{\pi-\gamma}{\omega_{sp}\sqrt{1-\zeta_{sp}^2}} \tag{51}$$

and the maximum percentage of overshoot (PO) is

$$\mathrm{PO} = 100\frac{A}{z_m}\exp\left[-\frac{(\pi-\gamma)\zeta_{sp}}{\sqrt{1-\zeta_{sp}^2}}\right] \tag{52}$$

Following the design specifications, ζ_{sp} and ω_{sp} are chosen to be 0.8 and 6.5 rad/s, respectively. By adjusting the location of the zero, it is found when z_m is decreased then it will reduce the peak time and increase the overshoot. When $z_m = 5$, the angle γ shown in Fig. 4 is 1.62 rad, and then $t_p = 0.39$ s and PO $= 10.21\%$. Thus, we choose the reference model as

$$M(s) = \frac{8.45(s+5)}{s^2+10.4s+42.25} \tag{53}$$

Based on the reference model and the design specifications with some conservative factors, the desired closed-loop bounds are chosen as illustrated in Fig. 5. Following the QFT design procedures, we have the robust boundaries for different frequencies as shown in Fig. 3. For satisfying the return difference equality, internal stability, and performance robustness, the nominal loop transfer function is chosen as

$$L_0(s) = \frac{25.7247(s+0.1)(s+0.7)(s+2.8)(s+4.1)}{s(s+1)(s+1.7)(s+3.2)(s-1.228)} \tag{54}$$

and the feedback controller will be

$$D(s) = \frac{1.9431(s+0.1)(s+0.7)(s+2.8)(s+4.1)(s+3.068)}{s(s+1)(s+1.7)(s+3.2)(s+0.884)} \tag{55}$$

As choosing $Q(s) = 1$, from Eq. (42), we solve $R(s)$:

$$R(s) = \frac{0.3464(-s^2)(10.24-s^2)(2.89-s^2)(1-s^2)(0.7815-s^2)}{(20.355-s^2)(9.4126-s^2)(8.0866-s^2)(0.4767-s^2)(0.0108-s^2)} \tag{56}$$

The required closed-loop eigenvalue locations are given by military specifications for flying qualities of piloted airplanes.[14] For the short-period mode, the characteristic equation of the closed-loop system is described by

$$s^2+2\zeta_{sp}\omega_{sp}s+\omega_{sp}^2 = 0 \tag{48}$$

the restricted range of damping ratio and natural frequency is

$$0.35 \le \zeta_{sp} \le 1.3 \qquad \omega_a \le \omega_{sp} \le \omega_b \tag{49}$$

where ω_a and ω_b depend on the FC and are given in Table 1.

In the following, we want to determine the corresponding prefilter. For unit step input signal tracking free of disturbance and noise,

$$\phi_{rr}(s) = 1/(-s^2), \qquad \phi_{dd}(s) = \phi_{nn}(s) = 0 \tag{57}$$

From Eqs. (13) and (14), we have

$$A(s) = \frac{(s+3.068)(s-1.228)}{(s+1)^2}, \qquad B(s) = \frac{-13.239(s+0.884)}{(s+1)^2} \tag{58}$$

$$V(s) = 1, \qquad U(s) = -\frac{(0.0121s+0.360)}{s+0.884} \tag{59}$$

From Eqs. (22) and (23), we have

$$\Delta_1(s) = \frac{0.5886(s+0.1037)(s+0.6905)(s+0.8840)(s+2.8457)(s+4.5733)(s+22.1831)}{(s+1)^2(s+4.5116)(s+2.8437)(s+0.6904)(s+0.1038)} \tag{60}$$

$$\Delta_2(s) = \frac{(s+3.068)(s+1.228)}{s(s+1)^2} \tag{61}$$

Since there exists only one unstable pole of the plant, we choose $\hat{F}(s)$ to be a constant. From Eq. (26), when $\hat{F}(s) = 0.6432$, $\hat{K}(s)$ is analytic in $Re(s) \geq 0$, and then from Eqs. (31) and (35), we have the prefilter

$$G(s) = -\frac{0.7869(s+0.1038)(s+0.6904)(s+1.0486)}{s(s^2+10.4s+42.25)(s+3.2)(s+1.7)(s+1.228)} \times \frac{(s+2.8437)(s+3.068)(s+4.5116)(s+5.6181)(s+19.0143)}{(s+1)(s+0.8840)} \tag{62}$$

In Fig. 2, by applying these two-degree-of-freedom compensators to four FC, the responses are shown in Figs. 5 and 6. The desired quantitative performance robustness linear quadratic optimal control is achieved.

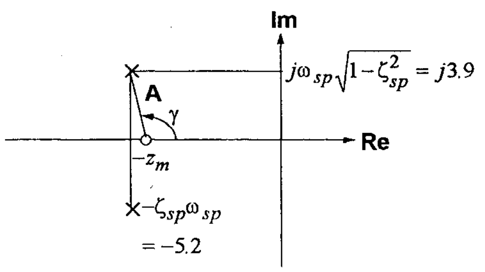

Fig. 4 Pole-zero pattern of the reference model in Eq. (53).

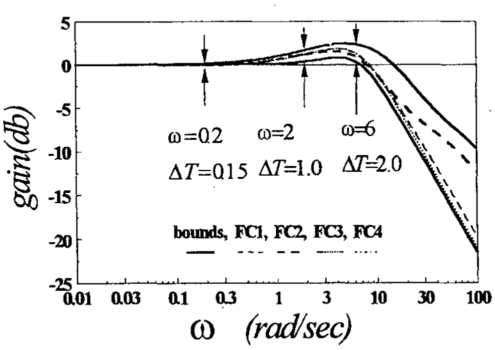

Fig. 5 Closed-loop system frequency response, q/q_c: bounds and response for four FC.

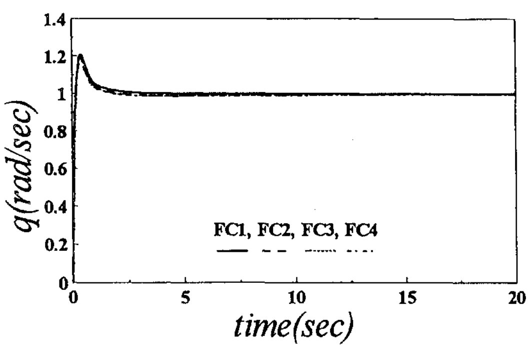

Fig. 6a Closed-loop system response to step command $q_e = 1$ rad/s: response for four FC.

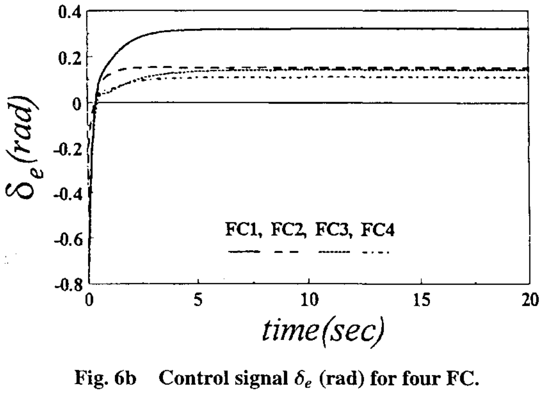

Fig. 6b Control signal δ_e (rad) for four FC.

VI. Conclusion

In this paper, the Wiener–Hopf optimal control and QFT is employed to derive the more generalized quantitative performance robustness optimal control system design algorithm. Since the internal stability is satisfied, this design algorithm can be applied to a broader class of problems. Two frequency-dependent weighting functions are shaped, and two-degree-of-freedom compensators are designed to achieve the quantitative performance robustness optimal control. For the minimum-phase plant, by appropriately factorizing the two-degree-of-freedom compensators, the feedback loop can be designed to satisfy the return difference equality.

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